Colorado's Risk-Limiting Audits and Ranked Choice Voting

By Chris Hughes

Post-election audits are used to determine whether the voting machines used in an election performed as they should, or if they malfunctioned (whether because of programming errors or malicious interference of a third party attempting to alter the results of an election). They are conducted after all ballots in the election have been counted but before an election is certified, in case the election must be recounted or rerun due to the results of the audit.

We have identified three types of post-election audits relevant to ranked choice voting elections: conventional audits, risk-limiting audits (RLAs), and Bayesian audits. This paper focuses on the application of risk-limiting audits to RCV races, particularly under Colorado law. In short, single-winner RCV races can currently be audited using risk-limiting audit methods, but running those audits requires computer science and audit expertise.[1] Multi-winner ranked choice voting

Risk-Limiting Audits

As stated in Colorado law, "risk-limiting audits provide a more effective manner of conducting audits than traditional audit methods in that risk-limiting audit methods typically require only limited resources for election races with wide margins of victory while investing greater resources in close races." Colo. Rev. Stat. § 1-7-515(1)(a). Colorado law also provides for local adoption of ranked choice voting, also known as instant runoff voting or the single transferable vote. Colo. Rev. Stat. § 31-10-617(1). Two cities in Colorado have adopted ranked choice voting for their local elections, and several are considering adoption, including Pueblo and Denver. Ranked Choice Voting Resource Center, Where Ranked Choice Voting is Used, https://www.rankedchoicevoting.org/where_used.

Ranked choice voting elections present challenges for risk-limiting audits, although ones that can be addressed in single-winner RCV elections. The biggest hurdle for risk-limiting audits of RCV elections is determining the margin of victory in a given RCV contest. To start, the margin in the final round of counting in an RCV race is not necessarily the closest margin that might have affected the outcome in an RCV contest. The simplest way of saying this is, if there is an earlier round where a candidate was eliminated and the margin between that candidate and the person with the next highest number of votes in that round is smaller than the margin in the final round, then that earlier round margin could be the actual margin you need to use to run a risk-limiting audit. The actual question being asked to determine RCV margins is more complicated and more difficult to answer than that, but that is the heart of the question.

Algorithms for determining the margin of victory in single-winner RCV races, also known as instant runoff voting (IRV), have been created and improved upon over the last five years.[2] The most recent and efficient algorithm for determining the margin of victory is described in

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[1] This description of the state of RLAs for RCV elections leaves out much of the detail required to run real-world RLAs. How to conduct audits across jurisdiction lines, how to randomly sample ballots, data security for ballot information, whether voting machine ballot data is available, and other detailed implementation questions for RLAs are not impacted by the voting method to be audited, and so are not covered here. This paper does not cover auditing multi-winner RCV, as there is currently no way to calculate margins for multi-winner RCV.

[2] Research in to creating these sorts of algorithms for top-two and top-three RCV (where only the top-two or top-three vote getters in the first round of a single-winner RCV race proceed to later rounds) is underway. Calculating margins will be simpler for these methods.

"Efficient Computation of IRV Margins" by Blom, Stuckey, Teague, and Nidhar, published in 2015. Software to produce the equations finding these margins is available at this github. It is written in the programming language C. The program must be run in tandem with CPLEX or similar software that can be used to run complex models such as the equations produced by the software linked to above. A simpler, rougher method of resolving this problem calculates the upper and lower bounds of the margin of victory across all rounds of counting and uses the lowest margin to calculate the sample size for the audit. Methods of estimation are laid out by David Cary in "Estimating the Margin of Victory for Instant-Runoff Voting."

Once the margin of victory in an RCV race is derived, that margin can then be plugged in to the risk-limiting audit formula laid out in "Conservative Statistical Post-Election Audits"[3] to determine how many ballots must be hand-counted to audit that RCV election. Typical RLA procedures are then followed: That many ballots must then be randomly selected from the paper ballot record for hand counting. Those ballots are checked against the digital record of ballots for discrepancies in the digital record. Depending on the number of discrepancies, the audit either confirms the result of the election or is run again, with a larger sample of ballots. If there are enough discrepancies in the ballot samples, a full manual recount may need to be conducted, as in any RLA.

Colorado's current RLA rule lays out RLA procedures specifically for the types of elections currently conducted in Colorado. The resources the state's RLA rule and procedures are based upon do not include procedures for ranked choice voting elections. The law as passed in Colorado, however, does not exclude the possibility of RCV audits, nor does it exclude the use of other auditing methods such as Bayesian audits (covered by Ron Rivest here), so long as those audits are truly risk-limiting. *See* Colo. Rev. Stat. § 1-7-515(5)(b). Also, the software developed for Colorado's risk-limiting audits is flexible, and can easily integrate other forms of audit.

Risk-limiting audit formulas that improve the efficiency of audits have been developed for vote-for-1 methods of election and are in use in Colorado. Risk-limiting audit formulas have not yet been designed specifically for ranked choice voting elections. The lack of RCV-specific formulas for RLAs means running the audit will require a lot of manual labor on the part of the auditor to set up programs to run the margin calculator and to run those margins through the older RLA formulas. The formulas are likely to require hand counts of more ballots than are strictly necessary for an RCV RLA, overcorrect for any discrepancies found in the course of the audit[4], and require more work on the part of the auditor to determine if/when the audit has satisfied the risk limit. Additionally, running an RLA on RCV right now requires a great amount of expertise with auditing procedures and computer science. While RLAs of single-winner RCV elections can be conducted, they are not easily accessible to most people. Tools that will improve the efficiency of RLAs for RCV elections are in preliminary stages of development now by Dr. Vanessa Teague of the University of Melbourne and other academics interested in risk-limiting audits of RCV.

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[3] RCV elections can currently only be audited by comparison audit, not ballot-polling audit, methods. A prospective RCV auditor must have access to the digital ballot record as well as the paper ballot record for an election.

[4] Which will, in turn, require relatively more ballots to be hand counted than would be required in an RLA process designed specifically for RCV.

Significant work remains to be done to get RLAs of RCV up and running. Further developing RLAs for RCV should follow a similar path as that followed for RLAs in Colorado:

- Phased implementation beginning with pilot programs
- Specifying the detail of audit procedures in rule, rather than in law.

Bayesian Audits

Bayesian audits are a form of audit that rely purely upon random samples of all ballots cast in an election. Unlike risk-limiting audits, they do not rely on the margin of victory to determine sample sizes when conducting the audit. This feature enables Bayesian audits to more readily be run on ranked choice voting elections (and other non-plurality elections). As with risk-limiting audits, however, the closer the margin of victory, the more likely a full recount may be required to effectively audit the election. "Bayesian Post-Election Tabulation Audits Explained and Extended", by Ron Rivest, last updated January 1, 2018, explains Bayesian audits in more detail.

A prototype Python script capable of running Bayesian audits is available for use on Ron Rivest's github. This script was written in order to audit the Australian Senate elections in 2016, but was ultimately not used by the Australian Election Commission. More about the development and potential application of this auditing method to Australian Senate elections is available in the paper "Auditing Australian Senate Ballots." The prototype has only been used with synthetic election data, not real-world RCV election data.

The script requires whoever wants to use it to run a Bayesian audit on an RCV election to have code, written in the coding language Python, capable of tabulating the results of a given RCV election according to the rules of the election being audited. The resources at the link do not provide that tabulation function. Python implementations of a variety of RCV methods are available at these links:

- https://code.google.com/archive/p/droop/
- https://github.com/DavidCary/CA-SB1288%09

The scripts available at the first link have not been reviewed by FairVote or other RCV experts. We are working on updating the Minneapolis code available at that archive to use the most recent counting rules adopted in Minneapolis. The second link is a Python implementation of California SB 1288, a local options bill passed by the California legislature in 2016 that would have permitted cities and counties in California to adopt single- or multi-winner RCV for local elections. Finally, in order to run a Bayesian audit, you will need access to the paper ballot record from an election.

Here is a proposed set of steps to follow when conducting a Bayesian audit of an RCV election. Procedure provided by Professor Ron Rivest, of MIT:

1. Pick an initial sample size (e.g. s = 100).
2. Draw s ballots at random from the set of cast ballots, and determine their contents by hand examination. (A "ballot" here is the full preference order on a ballot.)
3. Determine the "IRV/STV winner(s)" of the current sample.
4. If the computed winner(s) from (3) is not the same as the reported contest winner(s), increase the sample size (draw more ballots at random), determine their contents by

hand examination, and return to step (3). Repeat until step 3 does produce the reported contest winners.

5. On a laptop, compute 1000 "variants" of the sample. Each variant is a random subset of the sample, half the size of the sample. For example, if the sample has size 500, each variant would have size 250.
6. On the laptop, determine the "IRV/STV winner(s)" for each of the 1000 variant samples.
7. If 95 percent of the variants give the same IRV/STV winner(s) as the reported contest winner, stop and accept the reported contest winner as the correct winner.
8. Otherwise, increase the sample size (draw more ballots at random), determine their contents by hand examination, and return to step (3).[5]

Bayesian audits have not yet been proven to be a risk-limiting audit, though it is possible they can be truly risk-limiting. The procedure laid out above attempts to incorporate risk limiting in to the process in Step 7, by requiring 95% of the election variants to match the reported results of the election.

Research is underway to refine and improve the efficiency of Bayesian audits. To date, no real world election data has been subjected to this audit method. Additionally, as with RLAs of RCV, running a Bayesian audit requires computer science expertise and familiarity with statistical methods.

FairVote and the Ranked Choice Voting Resource Center are working with Ron Rivest and other auditing experts to get Bayesian audits for RCV elections fully up and running, using real election data. This, in hand with the work being done on risk-limiting audits, means ranked choice voting elections can soon be subject to the same rigorous auditing standards now in place in Colorado.

[5] The computer time required to run Step 6 is roughly equal to the time it takes to compute the most difficult-to-find margins for RCV using the algorithm mentioned in the Risk-Limiting Audits section, about two minutes. This is a simple coincidence, and may change as more efficient methods for both Bayesian audits and RLA margin computations are developed.

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